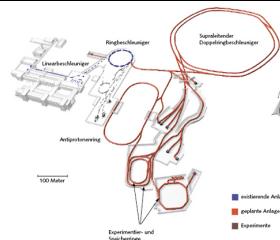
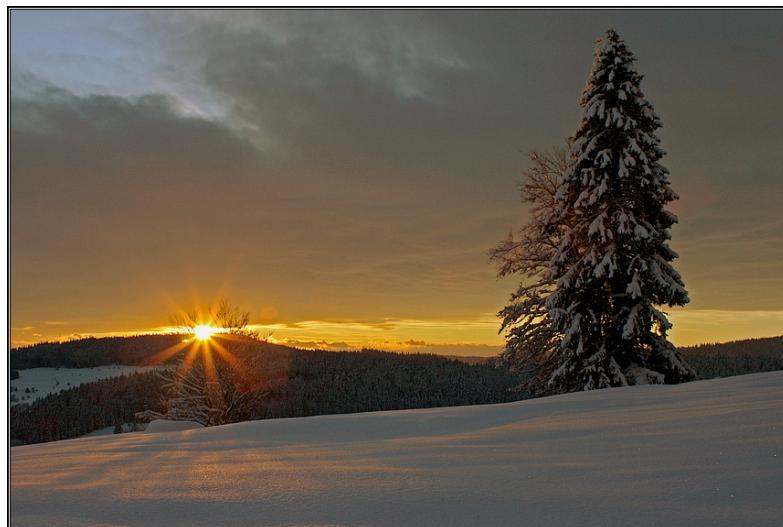


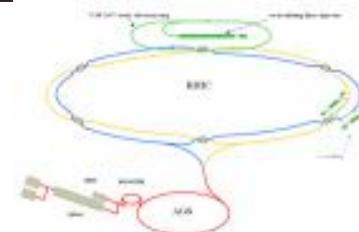


Phase transition dynamics in baryon-dense matter

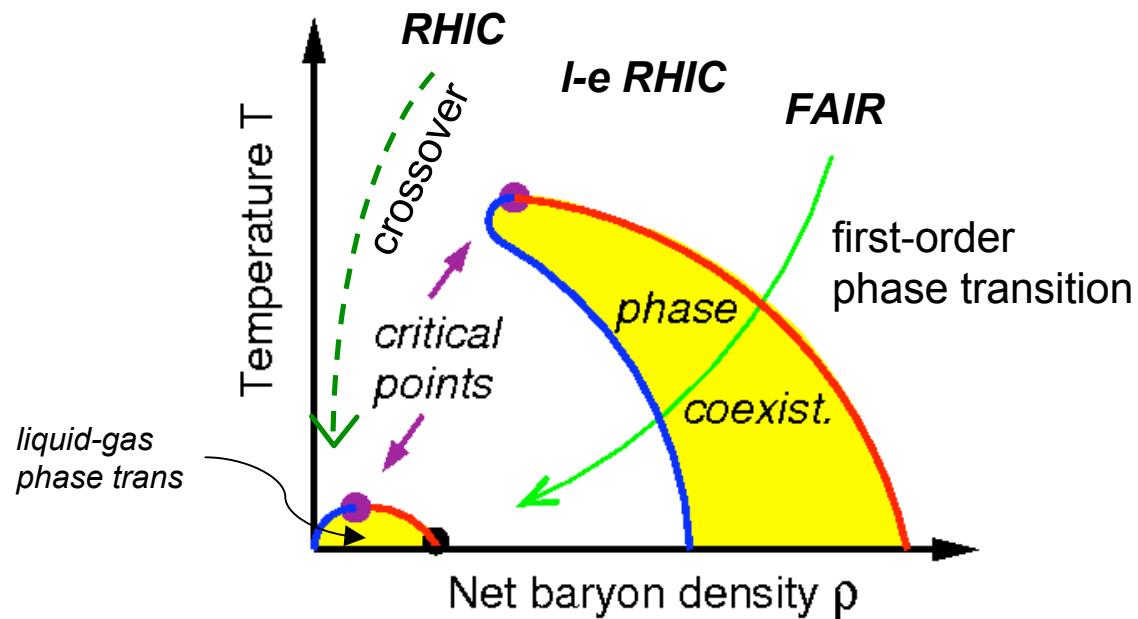
Jørgen Randrup



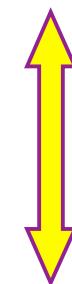
J. Randrup: CPOD-09



Exploring of the phase diagram of strongly interacting matter ...



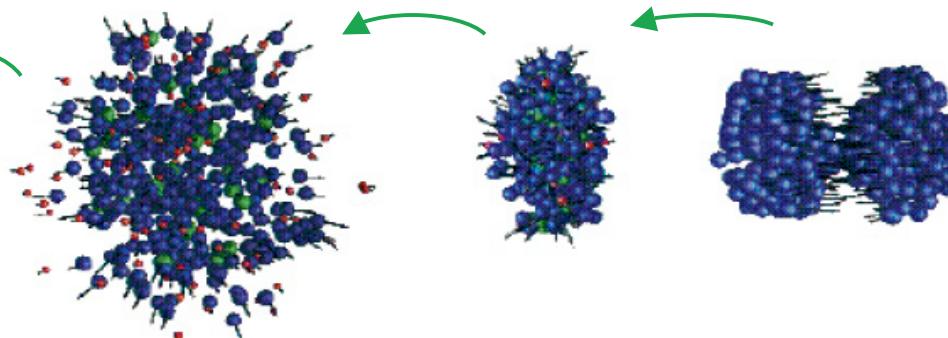
Large & old



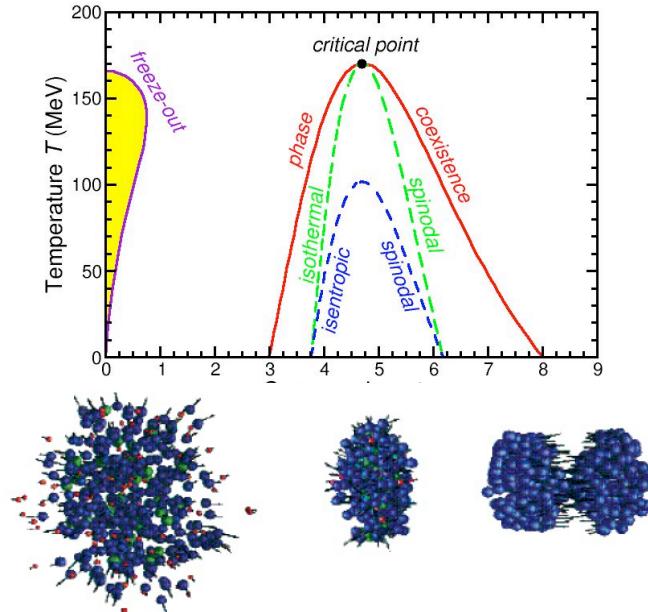
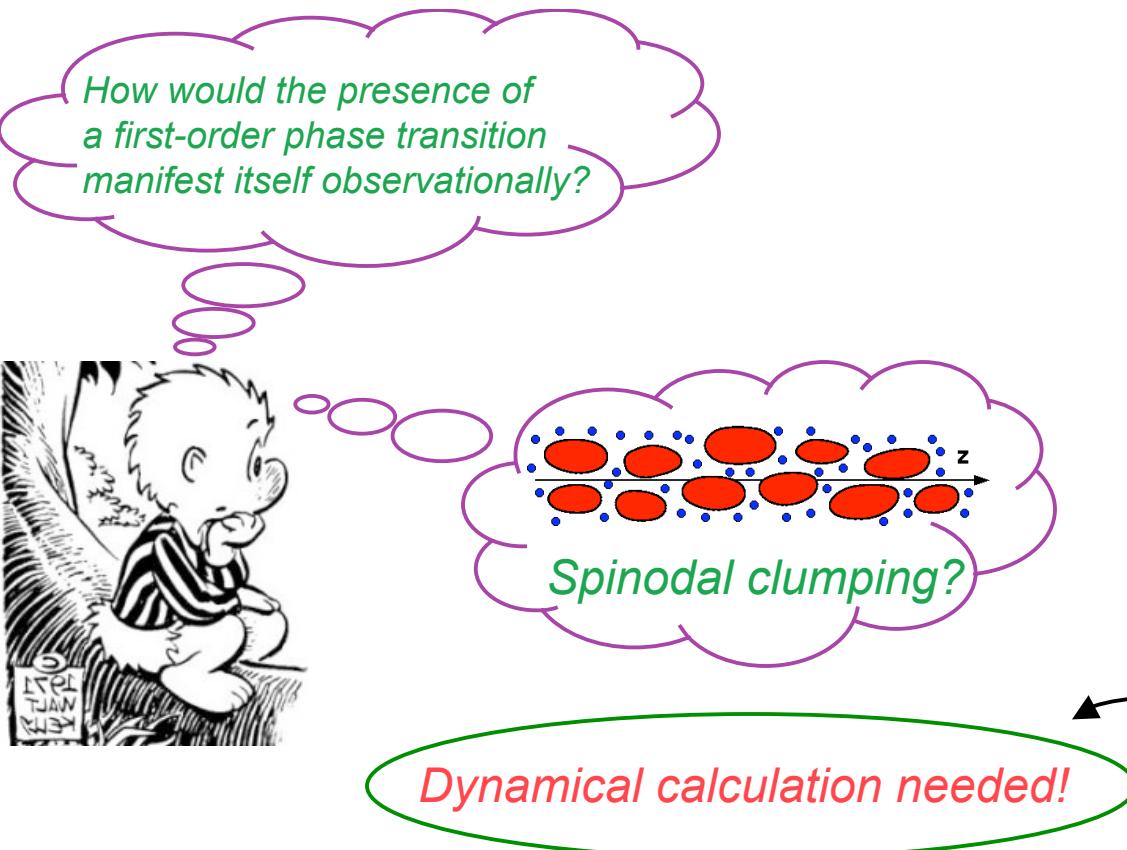
Small & young



RHIC
FAIR
NICA



J. Randrup: CPOD-09



REQUIREMENTS:

- EoS with phase transition $p(\rho, T)$
- Confined & deconfined phases
- Phase coexistence, incl interface*
- Spinodal modes in matter*
- Dynamics with instabilities*

* Requires finite range

Needed: Transport coefficients $\eta(\rho, T)$, $\xi(\rho, T)$, $\kappa(\rho, T)$

Important: Thermal conductivity κ !

JR, Phys. Rev. C79, 054911 (2009)

V.V. Skokov and D.D. Voskresensky: nucl-th/0811-3868;nucl-th/0903-4335

Equation of state: uniform matter

Finite range: gradients

Phase coexistence: interface

Collective modes: spinodal growth

Discussion

Uniform matter: Equation of State

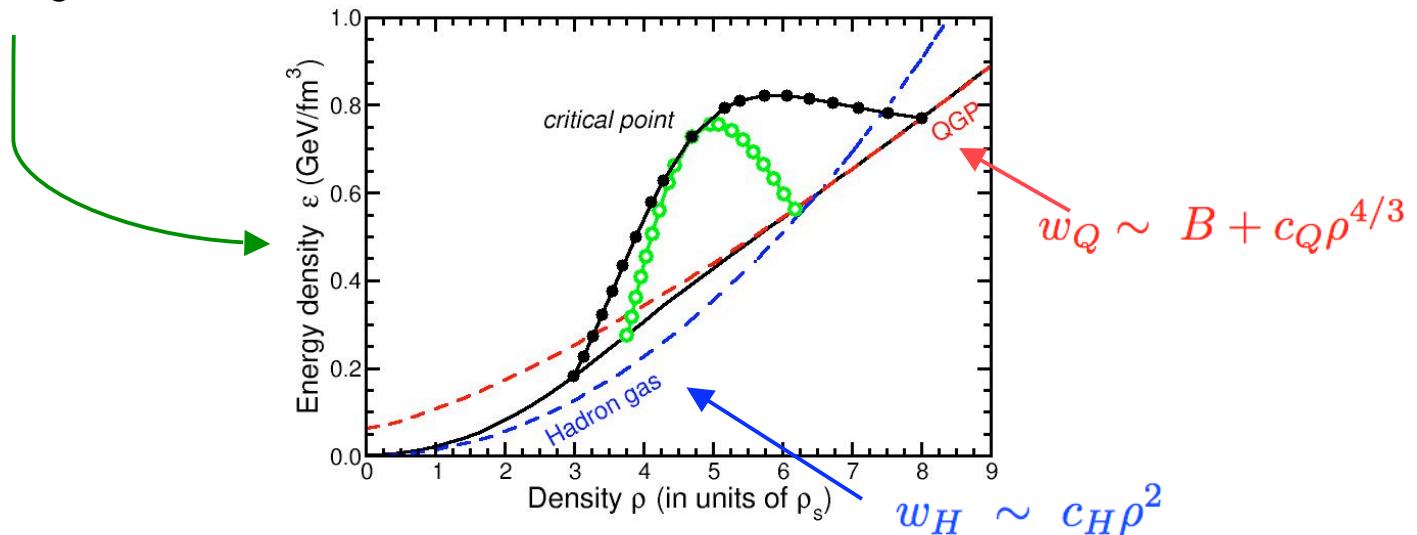


Construct simple temporary EoS:

$$\varepsilon = \kappa + w_0(\rho)$$

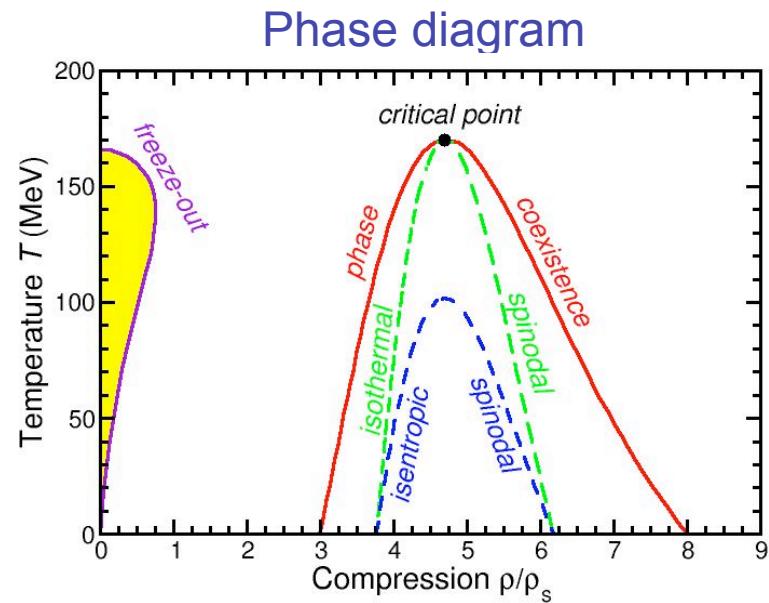
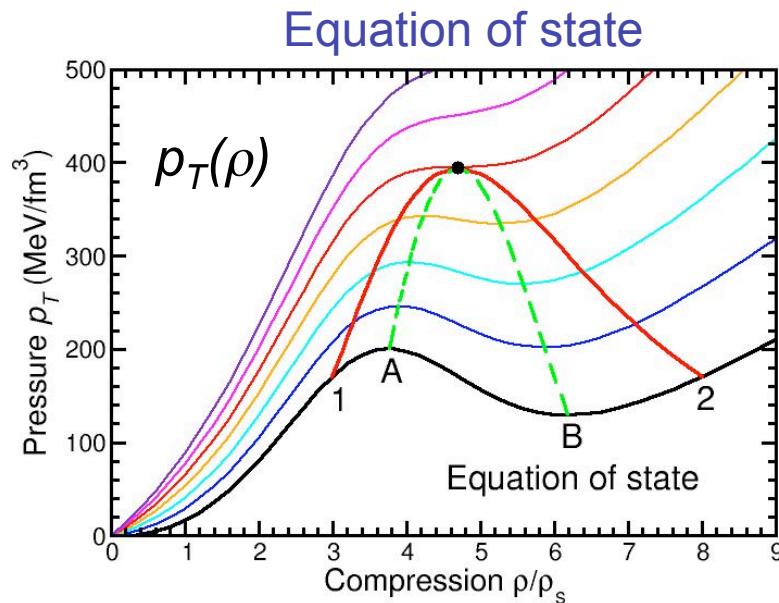
{ matter density ρ
energy density ε
kinetic energy density κ
compressional energy w_0

Interpolate between hadron gas and QGP



Uniform matter: phase coexistence

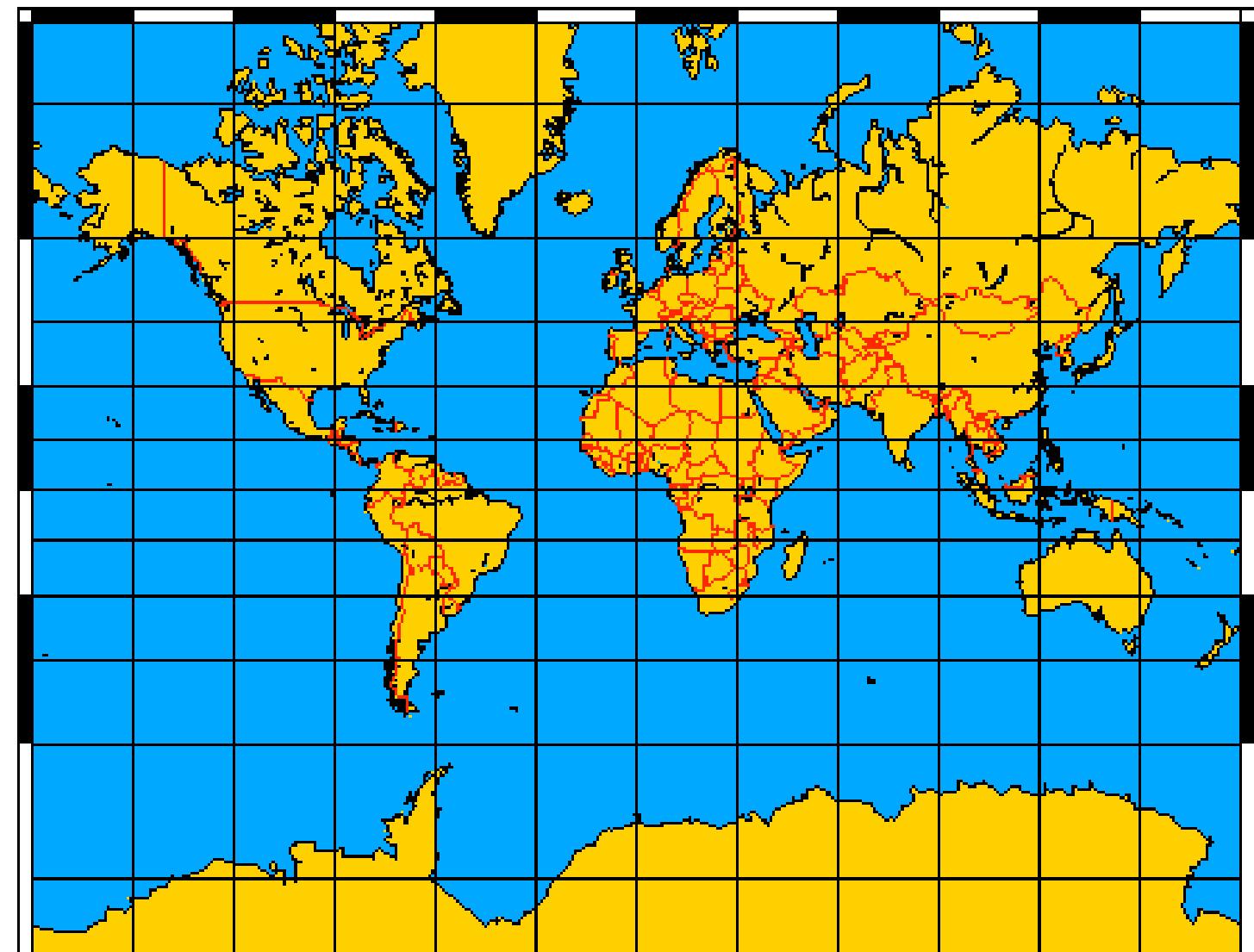
Phase coexistence: $\exists \rho_1 \neq \rho_2 : \mu_T(\rho_1) = \mu_T(\rho_2) \text{ & } p_T(\rho_1) = p_T(\rho_2)$



$T > T_c$: All phase points (ρ, T) are thermodynamically & mechanically stable

- $T < T_c$:
- | | |
|--|--|
| $\left. \begin{array}{l} \rho < \rho_1: \\ \rho_1 < \rho < \rho_A: \\ \rho_A < \rho < \rho_B: \\ \rho_B < \rho < \rho_2: \\ \rho_2 < \rho: \end{array} \right\}$ | Thermodynamically & mechanically stable |
| | Thermodynamically unstable but mechanically stable: <i>metastable</i> |
| | Thermodynamically & mechanically <u>unstable</u> $\partial_\rho p_T(\rho) < 0$ |
| | Thermodynamically unstable but mechanically stable: <i>metastable</i> |
| | Thermodynamic & mechanical stability |

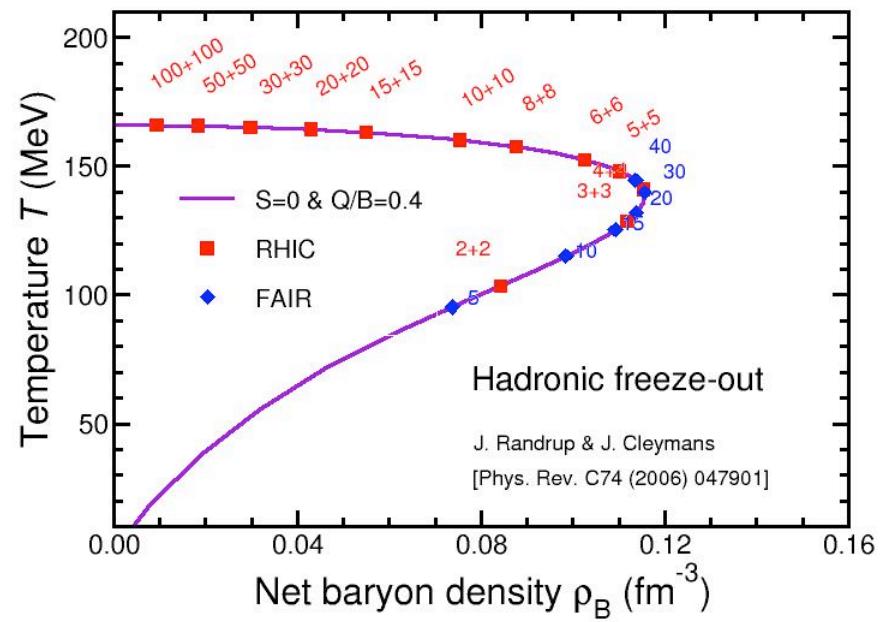
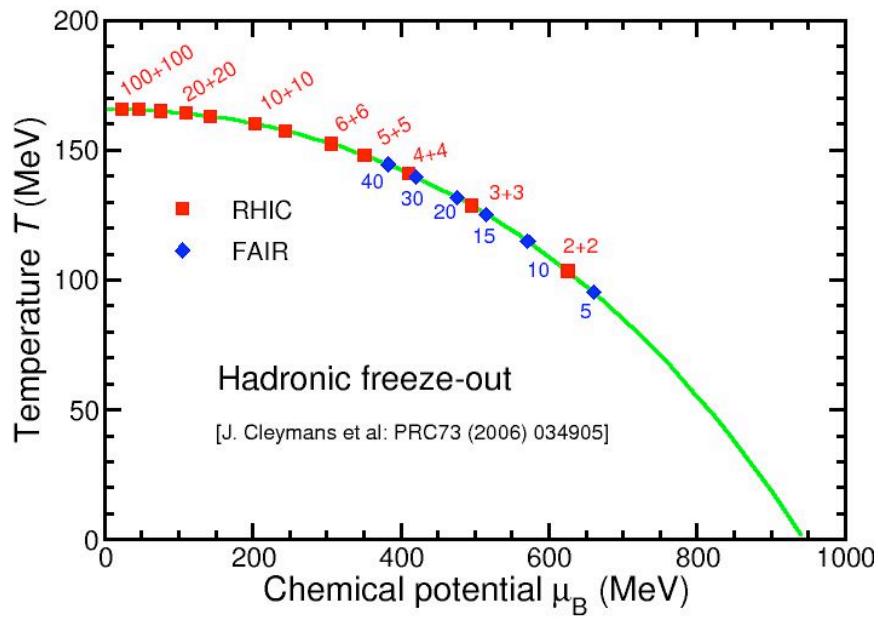
180° -150° -120° -90° -60° -30° 0° 30° 60° 90° 120° 150° 180°



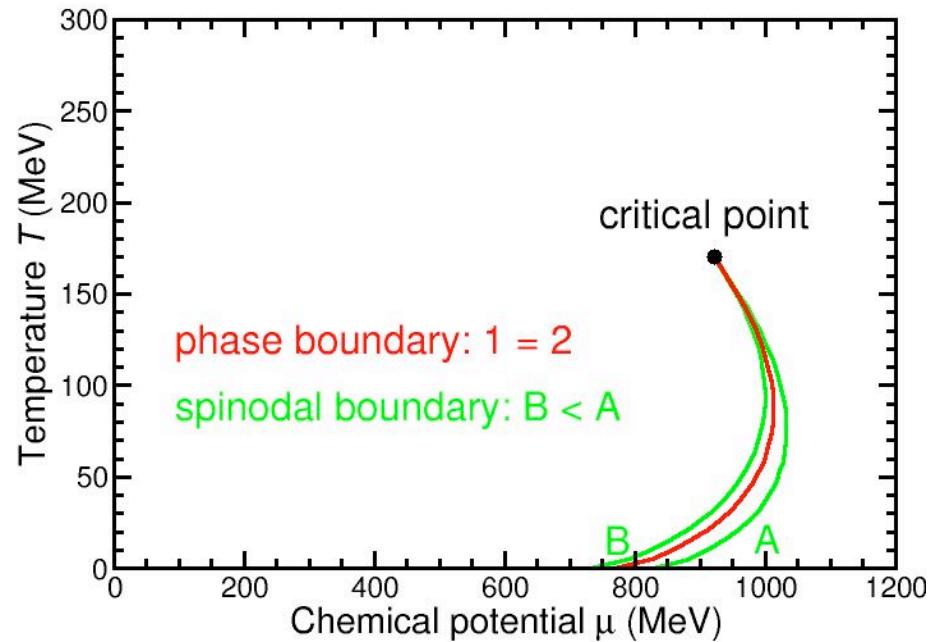
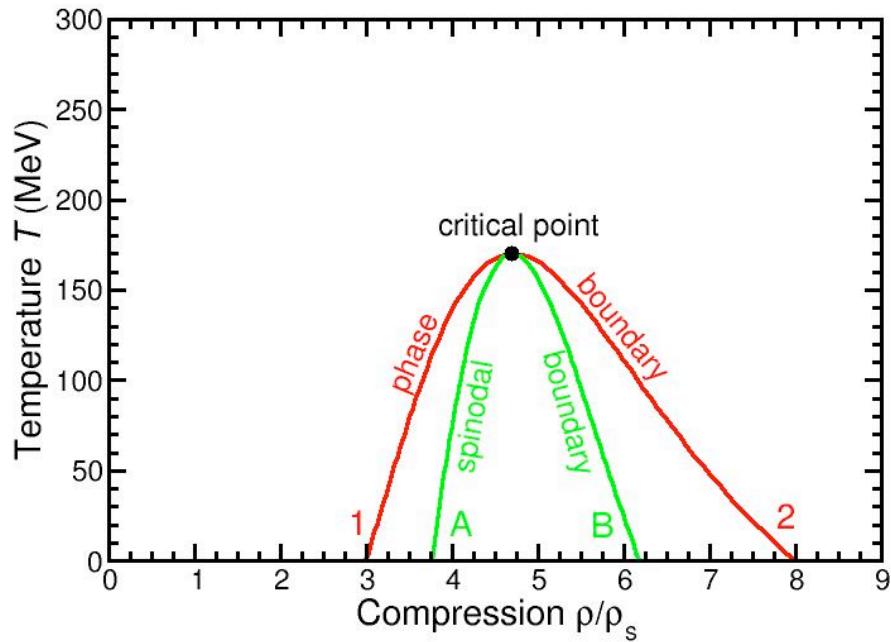
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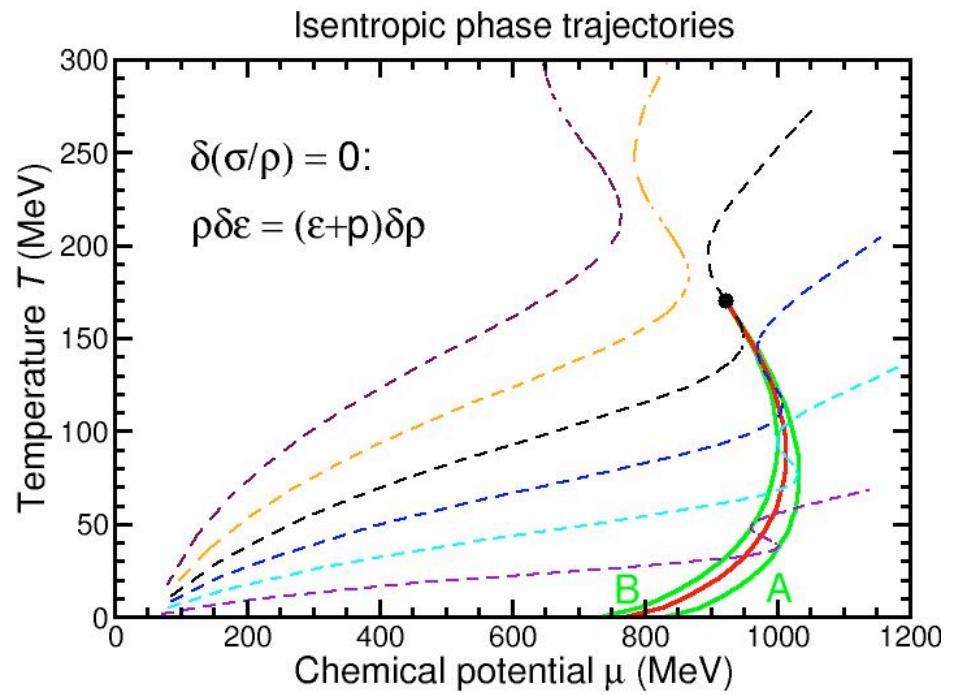
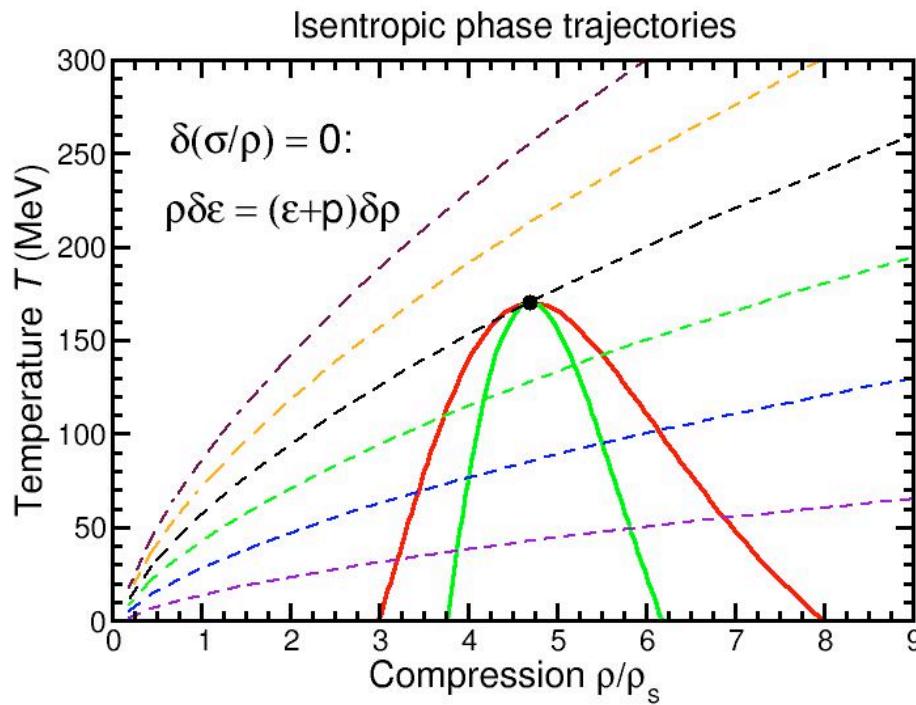
Example: Freeze-out line



Example: Phase diagram



Example: Phase trajectories



Non-uniform density $\tilde{\rho}(\mathbf{r})$



*gradient
correction*

$$\tilde{w}(\mathbf{r}) \equiv w_0(\tilde{\rho}(\mathbf{r})) + \frac{1}{2}C(\nabla \tilde{\rho}(\mathbf{r}))^2$$

local entropy density: $\tilde{\sigma}(\mathbf{r}) \equiv \sigma(\tilde{\varepsilon}(\mathbf{r}), \tilde{\rho}(\mathbf{r})) \Rightarrow \begin{cases} \tilde{\beta}(\mathbf{r}) \equiv \frac{\delta S}{\delta \tilde{\varepsilon}(\mathbf{r})} \Rightarrow \tilde{T}(\mathbf{r}) \text{ local} \\ \tilde{\alpha}(\mathbf{r}) \equiv \frac{\delta S}{\delta \tilde{\rho}(\mathbf{r})} \Rightarrow \tilde{\mu}(\mathbf{r}) \text{ local} \end{cases}$

\Rightarrow *total entropy:* $S[\tilde{\varepsilon}(\mathbf{r}), \tilde{\rho}(\mathbf{r})] \equiv \int \tilde{\sigma}(\mathbf{r}) d\mathbf{r}$

\Rightarrow *local pressure* $\tilde{p}(\mathbf{r})$ & *local enthalpy density* $\tilde{h}(\mathbf{r})$ & ...

$\Rightarrow \nabla \frac{\tilde{p}(\mathbf{r})}{\tilde{T}(\mathbf{r})} = -\tilde{\varepsilon}(\mathbf{r}) \nabla \tilde{\beta}(\mathbf{r}) - \tilde{\rho}(\mathbf{r}) \nabla \tilde{\alpha}(\mathbf{r})$: T & μ constant $\Rightarrow p$ constant

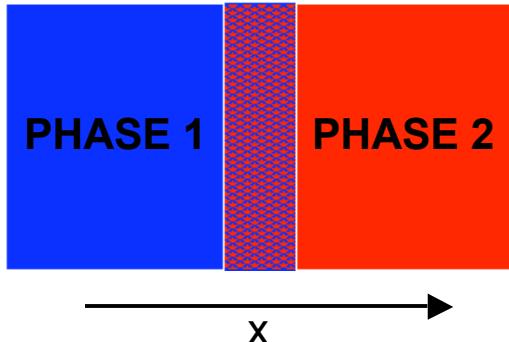
Canonical scenario: constant temperature T

local free energy density: $\tilde{f}_T(\mathbf{r}) = f_T(\tilde{\rho}(\mathbf{r})) + \frac{1}{2}C(\nabla \tilde{\rho}(\mathbf{r}))^2$

$$C = a^2 \varepsilon_g / \rho_g^2$$



Interface equilibrium



Global equilibrium requires constant T, μ, p :

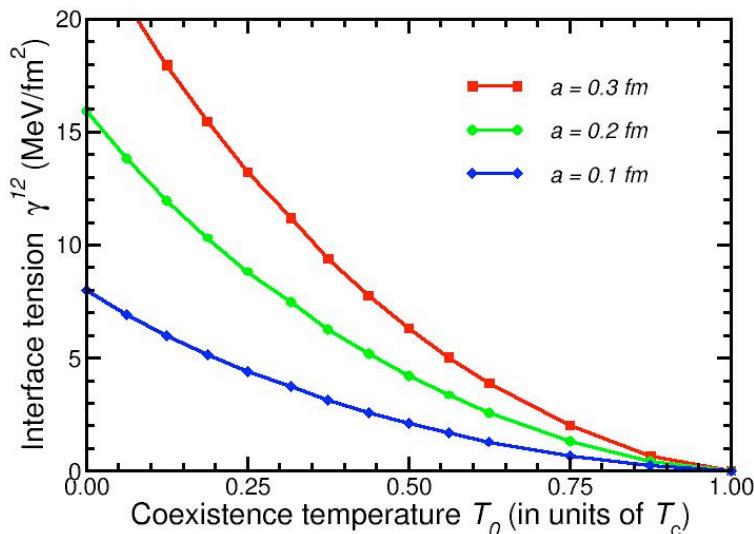
$$0 \doteq \delta S - \beta_0 \delta E - \alpha_0 \delta N = \int dx \left\{ [\tilde{\beta}(x) - \beta_0] \delta \tilde{\varepsilon}(x) + [\tilde{\alpha}(x) - \alpha_0] \delta \tilde{\rho}(x) \right\}$$
$$\Rightarrow \tilde{\beta}(x) = \beta_0 \text{ & } \tilde{\alpha}(x) = \alpha_0 \Rightarrow \tilde{p}(x) = p_0$$

[DG Ravenhall, CJ Pethick, JM Lattimer: NPA407 (1983) 571]

Equation for density profile: $C \partial_x^2 \rho(x) \doteq \mu_T(\rho(x)) - \mu_0 = \partial_\rho \Delta f_T(\rho(x))$

Interface-energy density: $\tilde{f}_{12}(x) = \tilde{f}(x) - f_i - \frac{f_2 - f_1}{\rho_2 - \rho_1} (\rho(x) - \rho_i)$

[WD Myers, WJ Swiatecki, CS Wang: NPA436 (1985) 185]



The interface tension is given by

$$\gamma_T^{12} = \int_{-\infty}^{+\infty} dx \tilde{f}_T^{12}(x)$$

$$\boxed{\gamma_T^{12} = \int_{\rho_1}^{\rho_2} d\rho [2C \Delta f_T(\rho)]^{\frac{1}{2}}}$$

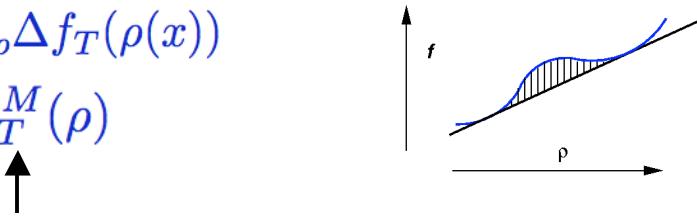
$[\rho(x) \text{ not needed!}]$

Interface density profile

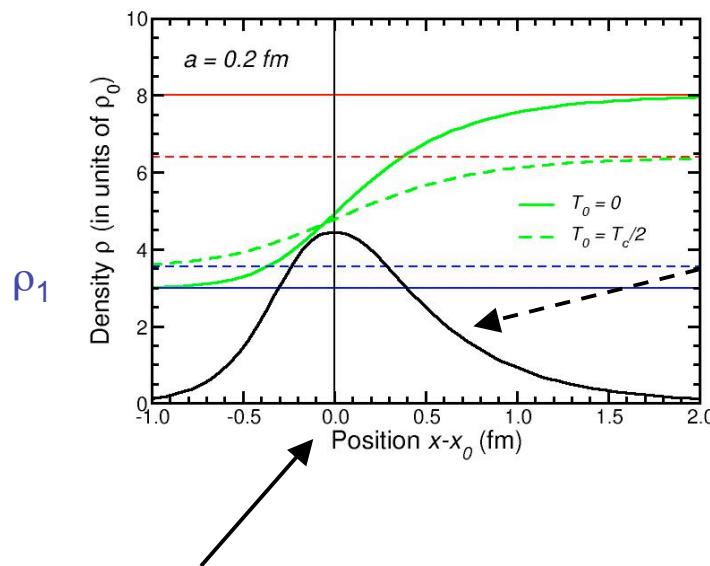
The interface density profile is determined by

$$C\partial_x^2\rho(x) \doteq \mu_T(\rho(x)) - \mu_0 = \partial_\rho \Delta f_T(\rho(x))$$

where $\Delta f_T(\rho) \equiv f_T(\rho) - f_T^M(\rho)$

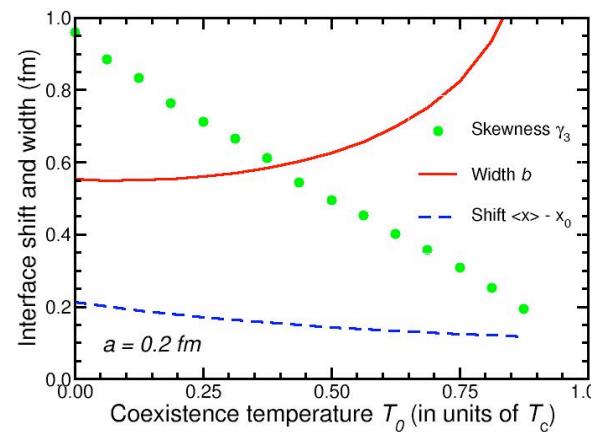


$$f_T^M(\rho) \equiv f_T(\rho_i) + \mu_0(\rho - \rho_i) \leq f_T(\rho)$$



$$x_0: \mu_T(\rho(x_0)) = \mu = \mu_T(\rho_i)$$

ρ_2
Interface location function:
 $g(x) = d\rho(x)/dx$



Location: $\langle x \rangle$

Width: b

Skewness: γ_3

$$b^2 = \langle (x - \langle x \rangle)^2 \rangle$$

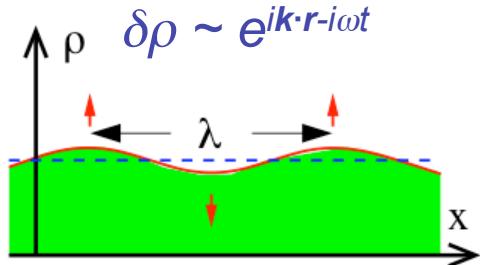
$$\gamma_3 = \langle (x - \langle x \rangle)^3 \rangle / b^3$$

Collective modes

Small undulations:

$$\delta\varepsilon(\mathbf{r}) = \tilde{\varepsilon}(\mathbf{r}) - \bar{\varepsilon}$$

$$\delta\rho(\mathbf{r}) = \tilde{\rho}(\mathbf{r}) - \bar{\rho}$$



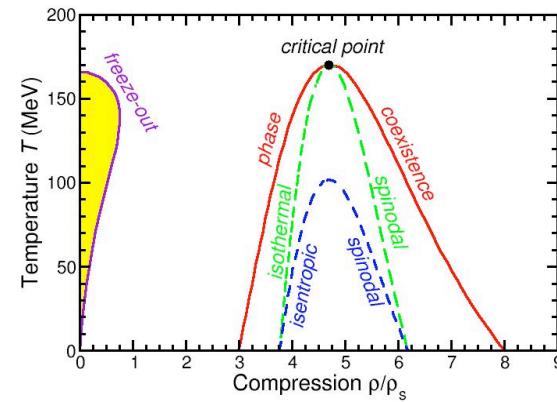
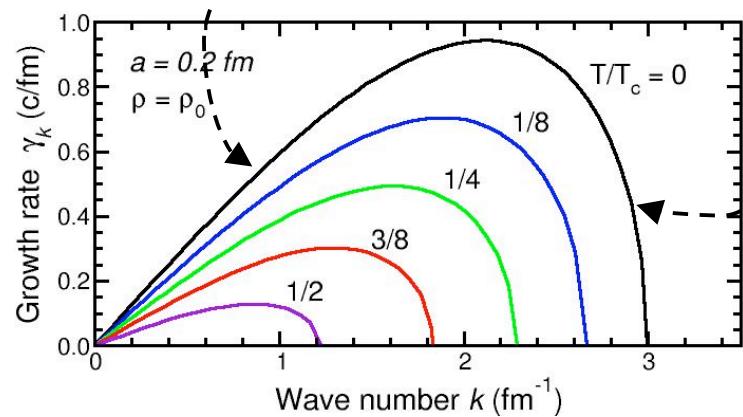
$$\tilde{p}(\mathbf{r}) \approx p(\tilde{\varepsilon}(\mathbf{r}), \tilde{\rho}(\mathbf{r})) - C\bar{\rho}\nabla^2\delta\rho(\mathbf{r}) \rightarrow \nabla^2\delta p(\mathbf{r}) \approx \frac{\partial p}{\partial \varepsilon}\nabla^2\delta\varepsilon(\mathbf{r}) + \frac{\partial p}{\partial \rho}\nabla^2\delta\rho(\mathbf{r}) - C\bar{\rho}\nabla^4\delta\rho(\mathbf{r})$$



Dispersion relation:

$$\omega_k^2 = v_s^2 k^2 + C \frac{\bar{\rho}^2}{\hbar} k^4 - i[\frac{4}{3}\eta + \zeta] \frac{k^2}{\hbar} \omega$$

gradient dissipation



Equations of motion:

$$\partial_t^2 \delta\varepsilon(\mathbf{r}) = \nabla^2 \delta p(\mathbf{r})$$

$$\bar{\rho} \partial_t \delta\rho(\mathbf{r}) = \bar{\rho} \partial_t \delta\varepsilon(\mathbf{r})$$

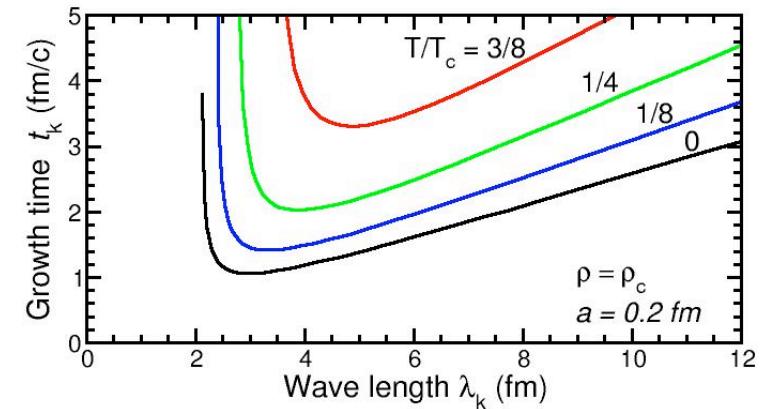
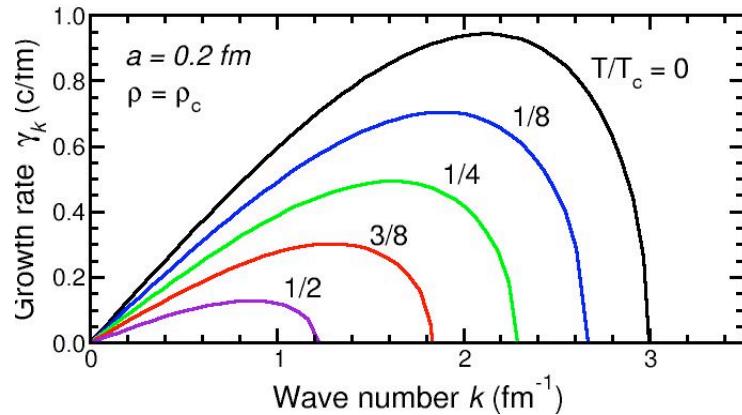
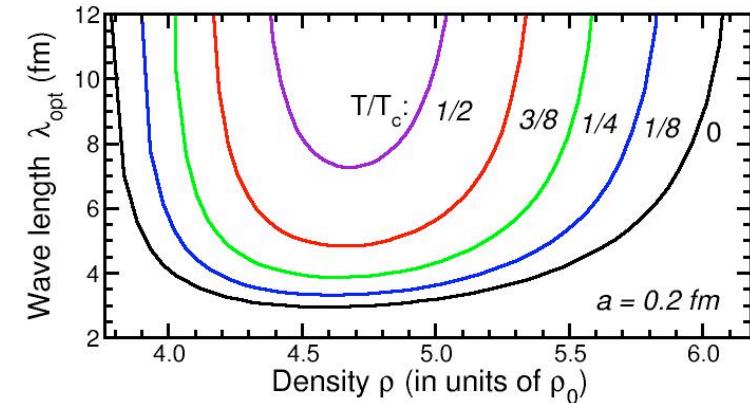
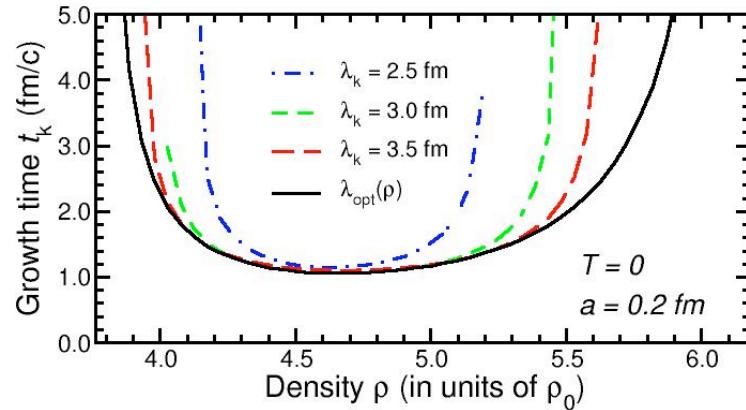
The frequency ω is imaginary inside the spinodal region: $\omega = i\gamma$

Spinodal region: $\omega_k = i\gamma_k$

$$\gamma_k = \pm \left[|v_s^2|k^2 - a^2 \frac{\varepsilon_g}{\bar{\varepsilon}} \frac{\bar{\rho}^2}{\rho_g^2} k^4 + \frac{1}{4} [\frac{4}{3}\eta + \zeta]^2 \frac{k^4}{\bar{h}^2} \right]^{\frac{1}{2}} - \frac{1}{2} [\frac{4}{3}\eta + \zeta] \frac{k^2}{\bar{h}}$$

$$v_s^2 = p_\varepsilon + \frac{\bar{\rho}}{\bar{h}} p_\rho = -\frac{\bar{T}}{\bar{h}} [\bar{h}^2 \sigma_{\varepsilon\varepsilon} + 2\bar{h}\bar{\rho}\sigma_{\varepsilon\rho} + \bar{\rho}^2 \sigma_{\rho\rho}] \quad (\text{isentropic})$$

Results for $\eta, \zeta = 0$:





Is there time for the phases to separate?

Thresholds in
collision energy:

E_C : critical point

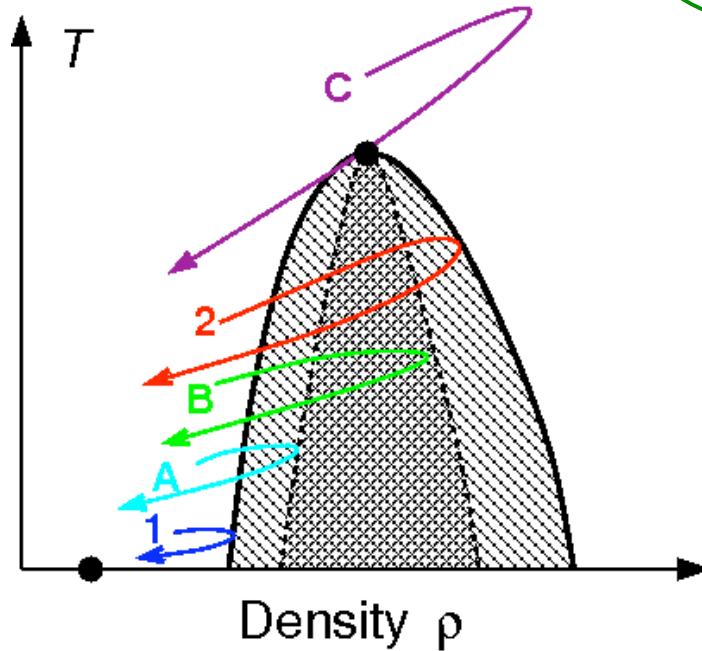
E_2 : higher boundary

E_B : higher spinodal

E_A : lower spinodal

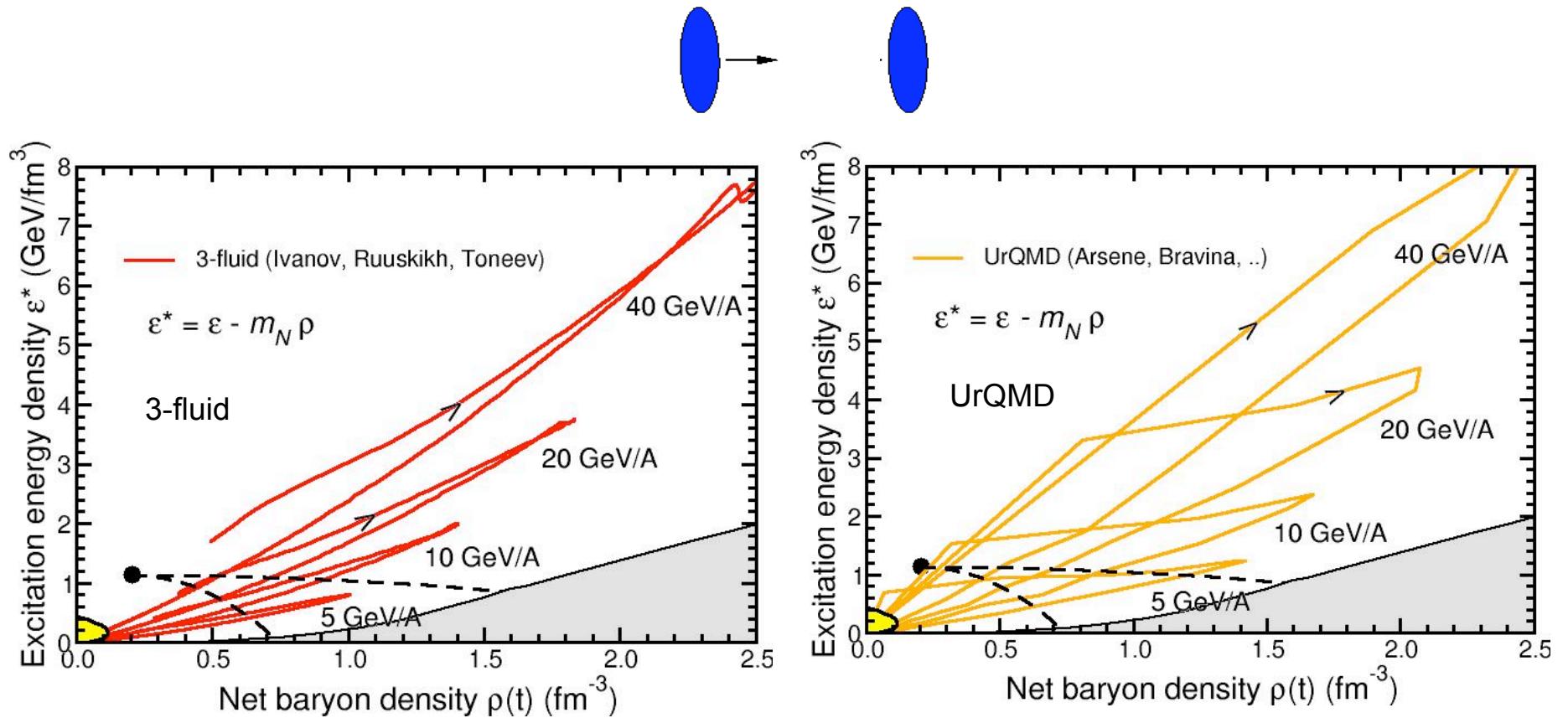
E_1 : lower boundary

Growth rate
versus
expansion rate?



Optimal conditions for spinodal phase separation: $E_B < E < E_2$

Dynamical phase trajectories (Au+Au, $b=0$):



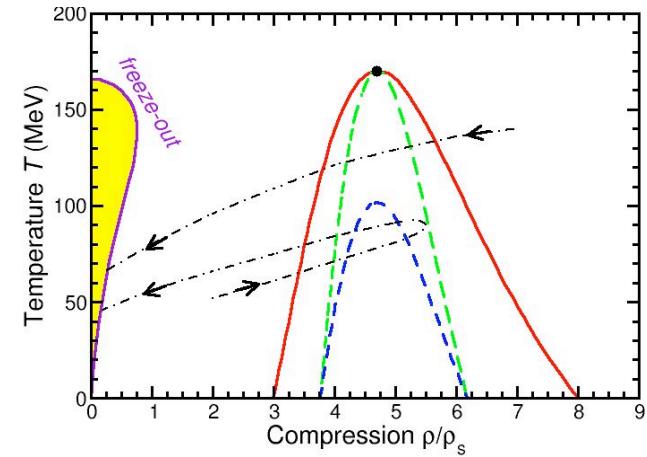
Arsene, Bravina, Cassing, Ivanov, Larionov, Randrup, Ruuskikh, Toneev, Zeeb, Zschiesche: PRC75 (2007) 034902

Phase-transition dynamics in baryon-dense matter:

Fluid dynamics with finite-range EoS

Need:

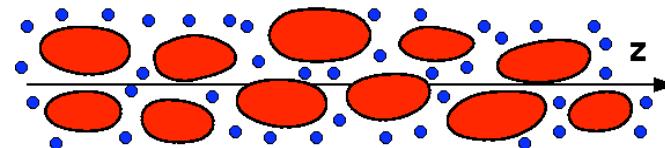
- Equation of state with finite range: $p(\rho, T)$
- Transport coefficients $\eta(\rho, T), \zeta(\rho, T), \kappa(\rho, T)$



*Thermal conductivity
must be included!*

Equations of motion

$\rho(\mathbf{r}, t), \varepsilon(\mathbf{r}, t), \mathbf{v}(\mathbf{r}, t)$



Critical Point and Onset of Deconfinement

5th International Workshop • June 8–12, 2009

Brookhaven National Laboratory, Long Island, New York, USA

TOPICS

- PHASE DIAGRAM OF QCD**
- DECONFINEMENT AND CHIRAL SYMMETRY RESTORATION**
- EQUATION OF STATE AND TRANSPORT PROPERTIES**
- CORRELATIONS AND FLUCTUATIONS**
- EQUILIBRATION AND HADRONIZATION**
- EXPERIMENTAL RESULTS FROM RHIC AND SPS**
- FUTURE EXPERIMENTS**

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